VALUE AND PRICE OF PRODUCTION: 
NEW EVIDENCE ON 
MARX’S TRANSFORMATION PROCEDURE*

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The main purpose of this article is to highlight a passage of the recently published Main Manuscript of Capital III¹ that was not included in Engels’ edition of this book. It is scarcely mentioned in the literature that Marx developed two examples of the transformation procedure in what became the chapter 9 of the third volume. Since Bortkiewicz published his famous articles on the “transformation problem”, Marx’s commentators have focused only the first example given in that chapter because it seemed compatible with that interpretation, the second example being thereby almost universally neglected. The missing text, which is a fundamental piece in the explanation of the transformation procedure, pertains to this second example and probably Engels’ omission contributed to the ultimate confusion regarding the transformation. In the first section, it is considered the first example of the transformation presented in chapter 9 and it is discussed its connection with the interpretation proposed by Bortkiewicz and his multiple followers. It will be shown that Bortkiewicz substantially altered the textual evidence in order to “adjust” it to his own interpretation. The second section reconstructs systematically the “second example”, focusing initially on the text omitted by Engels. The new evidence contained in the missing passage supports the interpretation of the transformation given by a series of authors since the 80s and shows that Marx’s procedure is logically consistent. The third section develops numerically the “second example” of the transformation procedure in both a static and a dynamic situation.

1. THE “FIRST EXAMPLE” IN CHAPTER 9 AND BORTKIEWICZ’S INTERPRETATION OF THE TRANSFORMATION

The extensive literature on the “transformation problem” has not examined the entire textual evidence regarding this aspect of Marx’s theory². The dominant interpretation —proposed by Tugan-Baranowsky and Bortkiewicz at the beginning of this century³— consists of a possible interpretation of

¹ Thanks to Paul Cockshott, Allin Cottrell, John Ernst, Alan Freeman, Paolo Giussani, Rolf Hecker, Andrew Kliman, Chai-on Lee, Fred Moseley, Adolfo Rodriguez and Alfredo Saad-Filho for helpful discussions. None of these people are responsible for the opinions I present in this work. This article is dedicated to Graciela.

² A non-exhaustive list of passages in which Marx deals with the transformation can be found in Ramos and Rodriguez [1996], p. 74, footnote 10. As far as I know, Marx presents five tabular illustrations of the transformation: The first (2 tables, 5 spheres) in Theories of Surplus Value II, pp. 67-68; the second (1 table, 4 spheres) in a letter to Engels dated 2 August 1862; the third (1 table, 4 spheres) in Theories of Surplus Value II, p. 389, the fourth (3 tables, 5 spheres) in Capital III, p. 255-6) and the fourth (1 table, 3 spheres), ibid., p. 264. The latter will be discussed in section 2.3.

³ Tugan [1905], pp. 170-4; Bortkiewicz [1906], [1907a] and [1907b]. Mühlfort [1893], [1895] presents the same approach and “solution” that Bortkiewicz but was only recently rediscovered by Howard and King [1989], p. 55-7.
one of the two examples presented in what Engels published as chapter 9 of volume III. In this chapter, however, there are two illustrations of the transformation procedure\(^4\), not only one as it is implicitly suggested in the literature. Certainly, if both examples had the same features, this would be irrelevant for the understanding of Marx’s presentation. Yet, this is not the case.

The core of the first example of the transformation procedure in chapter 9 is a set of two tables widely reproduced, with important modifications, by Marx’s commentators\(^5\):

<table>
<thead>
<tr>
<th>Capitals</th>
<th>Surplus-value</th>
<th>Value of commodities</th>
<th>Cost price</th>
<th>Rate of profit</th>
<th>Price of commodities</th>
<th>Rate of profit</th>
<th>Divergence of price from value</th>
</tr>
</thead>
<tbody>
<tr>
<td>I 80, + 20(_v)</td>
<td>20 100%</td>
<td>90</td>
<td>70</td>
<td>20%</td>
<td>90</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>II 70, + 30(_v)</td>
<td>30 100%</td>
<td>111 81</td>
<td>103</td>
<td>30%</td>
<td>111</td>
<td>81</td>
<td></td>
</tr>
<tr>
<td>III 60, + 40(_v)</td>
<td>40 100%</td>
<td>131 91</td>
<td>113</td>
<td>40%</td>
<td>131</td>
<td>91</td>
<td></td>
</tr>
<tr>
<td>IV 85, + 15(_v)</td>
<td>15 100%</td>
<td>70</td>
<td>55</td>
<td>15%</td>
<td>70</td>
<td>55</td>
<td></td>
</tr>
<tr>
<td>V 95, + 5(_v)</td>
<td>5 100%</td>
<td>20</td>
<td>20</td>
<td>5%</td>
<td>20</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>Total 390, + 110(_v)</td>
<td>110</td>
<td>422</td>
<td>312</td>
<td>22%</td>
<td>422</td>
<td>312</td>
<td></td>
</tr>
<tr>
<td>Average 78, + 22(_v)</td>
<td>—</td>
<td>22</td>
<td>—</td>
<td>22%</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
</tbody>
</table>

Since Bortkiewicz, these tables have been interpreted in the following way: Each table would correspond to a set of simultaneous equations, the first being a “system of values” and the second a “system of production prices”. In the “system of values”, the “value” of commodity “\(j\)” (\(\lambda_j\)) is defined as the sum of the value of constant capital used up (\(c^\lambda_{ij}\)) + the value of variable capital (\(v^\lambda_{ij}\)) + surplus-value (\(m_j\)), that is, as the sum of the value of cost-price + surplus-value, \(\lambda_j = (c^\lambda_{ij} + v^\lambda_{ij}) + m_j\). It is thus claimed that this is the definition of value presented in the first table; for example, the value of commodities produced in I would be (50\(_c\) + 20\(_v\)) + 20\(_s\) = 90. On the other hand, it is alleged that Marx calculates erroneously the prices of production in three steps: Firstly, in the first table, he would have obtained the “value rate of profit” as the quotient between total surplus-value and total advanced capital “in value”, 110\(_s\)/(390\(_c\) + 110\(_v\)) = 22%; then, transferred without modification the “cost-prices in value” (\(K^\lambda_{ij}\)) from the first to the second table, e.g. in sphere I, 50\(_c\) + 20\(_v\) = 70; lastly, obtained the prices of production by adding \(K^\lambda_{ij}\) + a proportion of total surplus-value calculated in accordance to the “value rate of profit”, (50\(_c\) + 20\(_v\)) + 22\(_p\) = 92.

Marx’s error would lie in the second step because when constructing the “system of production prices” he had “made the mistake of carrying over certain magnitudes [the so-called cost-prices in value, \(K^\lambda_{ij}\)] without alteration from the table of values into that of prices” whereas “in transforming

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\(^4\) Capital III, pp. 255-6 and p. 264; MEGA\(^2\) II/4.2, pp. 231-3 and pp. 240-1.

\(^5\) There are not conceptual differences between the Main Manuscript and the published version.

\(^6\) Superscript “\(\lambda\)” indicates that the magnitude corresponds to the value of inputs, and subscript “ij” that input “i” is used in the production of “j”. 
values into prices, it is inadmissible to exclude from the recalculation the constant and variable capital invested in the various spheres of production.”

In other words, Marx’s alleged procedure is incompatible with the (correct) specification of production price as the sum of the *price* of constant capital + the *price* of variable capital + profit, that is as the sum of the *price* of cost-price + profit, \( P_j = (c_i^j + v_i^j) + p_j = K^c_{ij} + p_j \). According to this interpretation, although “Marx recognized the need to transform input as well as output values into prices of production” he “was unable to extend his analysis to do so.”9

Then, Bortkiewicz’s “correction” consists of a specification of the “system of prices” in which the elements of the cost-price are “transformed” into production prices \( P_j = K^c_{ij} + p_j \), while the “system of values” remains as a set of equations in which the value is defined as \( \lambda_j = K^c_{ij} + m_i \). The widely known results of this algebraical setting are, firstly, that the “value rate of profit” and the “system of values” play no role in the determination of production prices10 and, secondly, that “it would not be permissible to equate total price with total value whilst simultaneously equating total profit with total surplus value”11.

In this interpretation, the value and production price of a commodity differ for two reasons: on the one hand, because surplus-value (\( m_i \)) is not equal to profit (\( p_j \)) and, on the other, because cost-price “in value” (\( K^c_{ij} \)) diverges from cost-price “in price” (\( K^p_{ij} \)). As a result of this, value and production price are defined in a *dualistic* fashion as two completely separated “systems” or “worlds”, thus severing the real and conceptual *unity* between them: neither value appears to be expressed effectively (although contradictorily) by price nor price is a manifestation of value. Both magnitudes are conceived, rather, as alternative, *non-related*, ideal rules of price formation12. In general terms, the dualistic understanding of the value/price relation neglects the *relation* between value-substance and value-form, i.e. the fact that the substance of value (abstract social labor-time) must appear as money13 and, at the same time, “money is labour time in the form of a general object, or the objectification of general labour time.”14 As Marx argues, the value/price relation is actually accomplished by means of *quantitative divergences*15, one of which is considered in the transformation of values into production prices.

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7 Bortkiewicz [1907a], p. 9. The argument had been already raised by a Böhm-Bawerk’s collaborator, J.v. Komorznyski: “Marx has disregarded the mutual dependence of the prices of the various products and the same omission is found in many passages where he presents the ‘price of production’ as ‘cost-price’ including profit but, at the same time, he defines ‘cost-price’ as the ‘value’ of the consumed constant and variable capital. [For example, Marx asserts that] prices of production ‘are equal to their cost elements (the value of the constant and variable capital consumed) plus a profit determined by the general rate of profit’. Komornslnski [1894], p. 294, 289 The cited passage from Marx is Capital III, p. 779. Komorznyski confuses the *value of constant and variable capital*, which is determined by the prices the capitalists pay when advancing money, with the *value of the means of production and consumption*, given by the social labor-time materialised in those commodities.

8 Superscript “P” indicates that the magnitude corresponds to the *production price* of inputs.

9 Howard and King [1975], p. 144.

10 Tugan [1905], p. 174; Samuelson [1970].

11 Bortkiewicz [1907a], p. 12.

12 Bortkiewicz, [1907a], pp. 6-7. See Ramos and Rodriguez [1996]. Dualism can be traced up to Engels’ “historical” conception of the transformation in which “value” would belong to a pre-capitalist stage (“simple commodity production”) while “production price” is particular to capitalism. Undoubtedly, this conception—also defended by Kautsky in the influential *Ökonomische Lehren*—gave way to Tugan and Bortkiewicz notion that “value” and “price” are two separated ideal rules of exchange. It is worth to note that Engels accepted Sombart’s opinion that “value... is not an empirical fact but an ideal or logical one”, Engels [1895], p. 1031, a vision also shared by Bernstein for whom value was merely a “logical construction”. On Engels and Kautsky “historic” interpretation of value, see Weeks [1981], pp. 12-23 and Hecker [1997].

13 “[m]oney as a measure of value is the necessary form of appearance of the measure of value which is immanent in commodities, namely labour-time.” Capital I, p. 188.

14 Grundrisse, p. 168.

15 “The magnitude of the value of a commodity... expresses a necessary relation to social labour-time which is inherent in the process by which its value is created. With the *transformation of the magnitude of value into the price* this necessary relation appears as the exchange-ratio between a single commodity and the money commodity which exists outside it. This...
As it has been mentioned, the dualistic authors claim that the definitions of value and price of production proposed by Tugan and Bortkiewicz are only an algebraical translation of the two tables of the first example in chapter 9. However, to be feasible, this opinion requires a substantial alteration of the textual evidence presented there. If—as it is alleged—both tables were respectively a “system of values” without prices and a “system of prices” without values, the latter must not contain columns for value and surplus-value. Yet, Marx’s second table does present those columns. This “detail” is never mentioned in the literature because it is usual to cite Bortkiewicz’s version of the tables in which those columns were suppressed in order to “adequate” the example to his dualistic conception of the transformation procedure. Additionally, it is clear that without suppressing the columns of value and surplus-value, the interpretation of both tables as “separated systems” is not plausible. Actually, Marx’s original presentation suggests that both tables are parts of a single-table example in which there are not the alleged difference between $K^\lambda$ and $K^p$ which is purported by Bortkiewicz and his followers. This point of view is notably buttressed by the second example of the transformation procedure to be considered in the following section.

2. THE “SECOND EXAMPLE”: A NON-DUALISTIC AND SEQUENTIAL ILLUSTRATION OF THE TRANSFORMATION PROCEDURE

In Engels’ edition of chapter 9, few pages after the example altered by Bortkiewicz, there is a second, different illustration of the transformation procedure. Although this example has exactly the same importance as the first one, it has remained “invisible” for the great number of authors dealing with the “transformation problem”. The confrontation between Engels’ edition of chapter 9 and the Main Manuscript permits one to establish two main differences: Engels omitted a relevant passage and probably added a numerical example, not developed in the original. As it will be shown, the omission reduced the strength of Marx’s presentation, contributing to the consolidation of Bortkiewicz’s interpretation. In this section, the “second example” is presented in 5 subsections following systematically Marx’s text.

2.1 Value and price of production in a passage omitted by Engels

In the Main Manuscript, the “second example” of the transformation procedure begins with a text included in Engels’ edition:

In Volumes 1 and 2 we were only concerned with the values of commodities. Now a part of this value has split away as the cost price, on the one hand, while the other, the production price of the commodity has also developed, as a transformed form of value.

relation may express both the magnitude of value of the commodity and the greater or lesser quantity of money for which it can be sold under the given circumstances. The possibility, therefore, of a quantitative incongruity between price and magnitude of value, i.e. the possibility that the price may diverge from the magnitude of value, is inherent in the price-form itself." Capital I, p. 196.

16 Compare Capital III, p. 256 and Bortkiewicz [1907a], p. 8. Bortkiewicz does not offer any explanation about the suppression of two columns as he does regarding the conversion of the tables into simple reproduction schemes. Although he complains about Marx alleged failure to “keep separate rigorously enough the two principles of value- and price-calculation.” (Ibid.), does not explain why these “principles” should be “rigorously separated”.

17 See footnote 4.

18 Capital III, p. 262; MEGA² II/4.2, p. 239, lines 37-40.
In the Main Manuscript, immediately after this text\(^{19}\), there is a passage omitted in Engels’ edition:

Der Kostenpreis ist, wie wir sehen, immer kleiner als der Werth der Waare. Der Produktionspreis kann kleiner, grösser oder gleich dem Werth der Waare sein. Der Werth der Waare = dem Werth des in der Production der Waare aufgezeherten Capitals plus dem Mehrwerth. Nehmen wir wie in derursprünglichen Entwicklung des Kostenpreises (Capitel I) Kostenpreis = Werth des in der Production der Waaren vorgeschossenen Capitals, so haben wir folgenden Gleichungen:

\[
Werth = \text{Kostenpreis} + \text{Mehrwert}. \quad W = K + m.
\]

oder Profit, als identisch
mit m.p.

\[
\text{Kostenpreis} = \text{Werth} – \text{Mehrwert}. \quad \text{oder } K = W – m.
\]

\[
\text{Produktionspreis} = \text{Kostenpreis} + \text{Profit}, \quad P = K + p'.
\]

berechnet nach der allgemeinen Profitrate = p’.
Da K = W – m und W = K + m, ist der Werth der Waare stets > als ihr Kostenpreis.
Je nachdem m oder p jeder besondren Productionssphäre gröser oder kleiner oder gleich, > < oder = dem durch die allgemeine Profitrate bestimmen Durchschnittsprofit, wird P > < = W.

Da W = K + m oder p, und P = K + p’ ist W = [P], wenn m = p’, > als P, wenn p’ < m und < P, wenn p’> m.\(^{20}\)

It is clear that Marx holds briefly in this passage that value = cost-price + surplus-value (W = K + m), or, rather, W = cost-price + produced profit (W = K + p), and that production price cost-price + appropriated profit (P = K + p’), where profit p’ is “calculated according to the general rate of profit” thereby differing from the produced profit, p\(^{21}\). The conceptual precision of these equations (Gleichungen) is not found anywhere in the published text of volume III. It is important to note, however, that similar formulations are found scattered in other places of the book. For example:

If we call the cost price K, the formula \(W = c + v + m\) is transformed into the formula \(W = K + m\), or commodity value = cost price + surplus-value... The capitalist cost price of the commodity is thus quantitatively distinct from its value or its actual cost price; it is smaller that the commodity’s value, for since \(W = K + m, K = W – m\).\(^{22}\)

Or, when referring to the production price of commodities of average composition:

\[^{19}\] Actually, after the above-cited text, there is a digression (in parentheses) referring to the calculation of production price when there is fixed capital. MEGA\(^2\) II/4.2, line 239-240, line 1-7.

\[^{20}\] Ibid., lines 7-27. In Marx’s original, the letter in square brackets is “K”, an obvious pen slip.


\[^{22}\] Capital III, p. 118. This text is not part of the Main Manuscript; notation modified according to MEW 25, p. 34, and that of the omitted passage.
As soon as this is the case ... the surplus-value \( v \) produces is equal to the average profit... because it is equal to the average profit, the price of production = cost price + profit = \( K + p = K + m \), which is equal in practice to the commodity’s value.\(^{23}\)

Now then, in the passage omitted by Engels it is possible to trace the modifications that the concept of value has underwent as a consequence of analyzing more superficial aspects of capitalist society. Firstly, value is no longer presented as the sum of \( c + v + m \) (which reveals its inner rationality) but as \( K + p \), cost-price + produced profit. Surplus-value has been qualitatively transformed into profit, i.e. it is no longer presented as generated by living-labor but in a mystified form, as produced by the whole advanced capital. However, the produced profit coincides quantitatively with surplus-value.\(^{24}\) Secondly, the sum \( c + v \) is also transformed qualitatively into a mystified magnitude, cost-price, in which there is no longer distinction between constant and variable capital. (In sub-section 2.4, it will be shown that, as produced profit \( p \), cost-price must also be transformed quantitatively.) Lastly, production price is presented as a mere quantitative transformed form of value, i.e. it differs from value only in magnitude. Therefore, the difference between \( W \) and \( P \) arises only from the difference between produced profit \( (p) \) and appropriated profit \( (p') \). In Marx’s equations, it is then clear that cost-price, \( K \), is the same magnitude for both, \( W \) and \( P \). This is at variance with the definition Bortkiewicz and his followers give for value and production price in which, as it has been seen in section 1, the cost-price in the “system of values” is different from the cost-price in the “system of prices” \( (K^\lambda \neq K^p) \).

2.2 The “hidden” table

After the missing text, there is a passage published by Engels in which Marx describes in words a table illustrating the transformation procedure:

If we take it that the composition of the average social capital is \( 80c + 20v \) and the annual rate of surplus value \( m' = 100 \) per cent, the average annual profit for a capital of 100 is 20 and the annual rate of profit is 20 per cent. For any cost price \( K \) of the commodities annually produced by a capital of 100, their price of production will be \( K + 20 \). In those spheres of production where the composition of capital is \( (80 – x)c + (20 + x)v \), the surplus-value actually created within the sphere, or the annual profit produced, is \( 20 + x \), i.e. more than 20, and the commodity value produced is \( K + 20 + x \), more than \( K + 20 \), or more than the price of production. In those spheres where the composition of capital is \( (80 + x)c + (20 – x)v \), the surplus-value or profit annually created is \( 20 – x \), i.e. less than 20, and the commodity value therefore \( K + 20 - x \), i.e. less than the price of production, which is \( K + 20 \). Leaving aside any variation in turnover time, the production prices of commodities would be equal to their values only in cases where the composition of capital was by chance precisely \( 80c + 20v \).\(^{25}\)

The “hidden table”, made up from 3 spheres of average, low and high composition, is easy to be written down:

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\(^{23}\) Capital III, p. 309, MEGA\(^2\) II/4.2, p. 284, lines 3-6. There are minor modifications in the published version.

\(^{24}\) “Profit is firstly only another name, or another category, for surplus value. Since the whole labour appears to be paid through this form of wages, the unpaid part of the same necessarily appears as being not a result of labour, but a result of capital, and not of the variable part of the same, but of the total capital. The surplus value thereby receives the form of profit, without any quantitative difference between the one and the other. It is only the illusory form of appearance of the same.” Letter to Engels, 30 April 1868.

This table is in direct connection and is consistent with the equations of value and production price omitted by Engels. Value = cost-price + produced profit and production price = cost-price + appropriated profit; there are not two cost-prices, one for the “system of values” and other for the “system of prices”, but only one, common magnitude for both, values and production prices. In fact, the “hidden table” is simply a semi-algebraical illustration of the preceding formulation assuming that the composition of average capital is $80c + 20v$, a rate of surplus value equal to 100% and no fixed capital. Yet, this connection between Marx’s equations and their subsequent illustration was lost in Engels’ edition. It is important to note that this presentation involves a single table, thus differing from the “first example” in which there are two seemingly separated tables and, of course, from Bortkiewicz’s altered version of it commonly used as “textual evidence” to discuss the “transformation problem”. On the basis on Marx’s equations and on the single table, the “double equality” ($\sum$ values = $\sum$ prices of production and $\sum$ surplus-values = $\sum$ profits) is easily obtained since values and production prices differ only because surplus-values differ from profit, being K the same for both W and P.

### 2.3 The arithmetical example, an Engels’ addition?

As, assuming a uniform rate of surplus-value, the differences between values and production prices are determined by the composition of capital (C/V), once Marx’s constructs the “hidden table” he undertakes the explanation of this ratio. The point is summarized in a sentence published by Engels with modifications:

> Wenn m + n constante Grössen = 100, x irgend eine beliebige variable Grösse; wenn die Zusammensetzung des gesellschaftlichen Durchschnittscapitals = $C^m C^n$, so sind Capitalien con der Form $C^{m+x} V^{n-x}$ Capitalien von höherer, dagegen Capitalien con der Form $C^{m-x} V^{n+x}$ Capitalien von niedriger organischer Composition.

26 MEGA² II/4.2, p. 241, lines 24-28; the modified text in Capital III, p. 264.

27 Capital III, p. 264.

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<table>
<thead>
<tr>
<th></th>
<th>Cost-price</th>
<th>Profit</th>
<th>Price of production</th>
<th>Surplus-value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$K = 80c + 20v = 100$</td>
<td>20</td>
<td>$K + 20 = 120$</td>
<td>20 + $K + 20 = 120$</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>$K = (80- x)c + (20+ x)v = 100$</td>
<td>20</td>
<td>$K + 20 = 120$</td>
<td>20 + x</td>
<td>$K + 20 + x = 120 + x$</td>
</tr>
<tr>
<td>3.</td>
<td>$K = (80+ x)c + (20- x)v = 100$</td>
<td>20</td>
<td>$K + 20 = 120$</td>
<td>20 - x</td>
<td>$K + 20 - x = 120 - x$</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>3K = 240c + 60v = 300</td>
<td>60</td>
<td>3K + 60 = 360</td>
<td>60</td>
<td>3K + 60 = 360</td>
</tr>
</tbody>
</table>

Although this illustration was probably inserted by Engels, it is simply an arithmetical version of the “hidden table” assuming $x = 10$. Moreover, the same example is found in a letter to Engels, dated 30 April 1868:

> Now this [the mass of capital belonging to each sphere of production] seizes a certain part of the total surplus value, in that proportion in which it forms a part of the total
social capital] is only achieved if the annual output of commodities is each sphere of production (...the total capital = 80c + 20, and the social rate of profit = 20/(80c + 20)) is sold at the cost price + 20% profit on the invested value of capital (regardless of the amount the invested fixed capital which enters, of does not enter, the annual cost price). But the determination of the price of the commodities must also diverge from their values. Only in those spheres of production where the percentage composition of capital is 80c + 20, does the price K (cost price) + 20% of the invested capital coincide with their value. Where the composition is higher (e.g. 90c + 10v) this price is above their value, where the composition is smaller (e.g. 70c + 30v), under their value.

Regarding the version published by Engels’, it is clear that although the example does apply Marx’s equations in the passage previously omitted, they are not explicitly presented as they were in the Main Manuscript. As a result, the numerical example lost its conceptual explanation and has been consequently ignored. Moreover, as Engels did not publish this illustration as a table but as a part of the text, it is widely believed that in chapter 9 there is only one tabular example of the transformation procedure.28

2.4 What is the cost-price, K?

As it has been seen, in the passage omitted by Engels, as well as in the “hidden table” and in the subsequent inserted arithmetical illustration, the cost-price of a commodity, K, is the same magnitude for both, its value and its production price. But, what is the cost-price? Does it correspond to the value or to the price of production of the inputs which the capitalist purchases at the beginning of the cycle? Bortkiewicz’s interpretation of the “first example” of chapter 9 is grounded on the belief that K corresponds to the value of those commodities. This opinion gave rise to the charge that Marx “completed the half the process of transformation” because although he transformed the outputs, left the inputs (i.e. the cost-prices) “in value terms.”29 However, insofar as in the “second example” is the followed in its original sequence, starting with the equations in the missing passage, the charge against Marx looses any solid ground. In effect, after considering the capital composition, Marx grapple with this issue, focusing on the effect that the transformation has on the category of cost-price. Then, he writes:

Es ist durch die jetzt gegebene Entwicklung allerdings eine Modification eingetreten in respect to the determination of the cost price of commodities. Ursprünglich angenommen, daß der Kostpreiß einer Waare = dem Werth der in ihrer Production consummirten Waaren. Da aber der Productionspreiß einer Waare als Kostpreiß in die Preißbildung einer anderen Waare eingehend die Productionsmittel gebildet wird. Es ist nöthig sich dieser modifizierten Bedeutung des Kostpreisses zu erinnern und sich daher zu erinnern, daß wenn in einer besonderen Productionsphäre der Kostpreiß der Waare und der Werth der in ihrer Production consummirten Productionsmittel gleichgesetzt werden, stets ein Irrthum

28 Note also that, in the “hidden table”, spheres 2 and 3 exhibit, respectively, a low and high composition of capital while in the arithmetical illustration this order was inverted.

29 Howard and King [1975], p. 144.
In this passage, included with modifications by Engels, Marx gives a clear answer regarding the determination of cost-price: K does correspond to the price of production of the commodities purchased by the capitalists, not to their values: “There is always the possibility of an error if, in any particular production sphere, the cost-price and the value of the consumed means of production are equated.” Therefore, K corresponds to the production price of inputs. (It is important to note, however, that this is only a particular case in which all capitalists would obtain the general rate of profit while, in general, K is determined by the market price of the inputs. In this article, it is assumed that commodities are exchanged for their prices of production.)

The published version of the above-quoted passage has been frequently cited out of context as a “proof” that Marx “was aware that he had left the inputs in value terms.” However, as it is read in connection with the equations for W and P included in the text omitted by Engels, it is clear that Marx is simply stating that cost-price has undergone a quantitative modification which should be added to other “modifications” already considered, namely, the transformation of surplus-value into profit, the transformation of constant capital + variable capital into cost-price and the transformation of value into price of production (see 2.2). Now, Marx argues, the cost-price is also transformed quantitatively, not only qualitatively, because it is determined by the prices of production, not by the values, of the inputs.

The quantitative modification of cost-price has an important implication for the determination of the magnitude of value, W: The money-value advanced by capitalists no longer corresponds to the value of the inputs, but to their production price. Constant capital, i.e. the social labor-time transferred by the means of production to the value of commodities is not the labor-time objectified in the means of production but the labor-time represented by the money that the capitalists advance at the start of the cycle, determined by their prices of production. In the same way, variable capital is not the value crystallized in wage goods but the labor-time represented by the price of production of those commodities. Therefore, the fraction of value corresponding to K is given by the production prices of the inputs; in other words, the portion of the labor-time socially necessary to produce a commodity corresponding to the inputs is the labor-time represented by their money production prices. According to Marx’s equations in the passage omitted by Engels, the fraction of value K is also a component of the

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31 “Let us... assume that the productivity of the spinner’s labour... remains constant... but that the exchange-value of the cotton varies, either by rising to six times its former [price] or by falling to one-sixth of that [price]. In both these cases, the spinner... adds as much value, as he did before the change in the value... Nevertheless, the value he transfers from the cotton to the yarn is either six times what it was before, or, in the second case, one-sixth as much. The same result occurs when the value of the instruments of labour rises or falls, while their usefulness in the labour process remains unaltered.” Capital I, pp. 309-10. In Penguin’s edition, German word “Preis” was erroneously translated as “values”. See MEW, 23, p. 216.

32 For example, Sweezy [1942], p. 115.

33 “...the difference between [production price] and value, insofar as it enters into the price of the new commodity independently of its own production process is incorporated into the value of the new commodity as an antecedent element.” Theories of Surplus Value III, p. 167. In this book, Marx refers to production price as “cost-price”, term which has been replaced; emphasis added.

34 “As for the variable capital, the average daily wage is certainly always equal to the value product of the number of hours that the worker must work in order to produce his necessary means of subsistence; but this number of hours is itself distorted by the fact that the production prices of the necessary means of subsistence diverge from their values.” Capital III, p. 261.

35 Values and production prices are measured in both, social labor-time and money. The money that capitalists advance is only an objective representation of social labor-time.
production prices, so that values and production prices differ only because the produced profit (p) is not equal to the appropriated profit (p').

The quantitative modification of cost-price permits one to re-write K as follows:

\[ K = \text{value of inputs} \pm \delta \]

where \( \delta \) stands for the divergence between the value and the production price of the inputs. Therefore, \( W \) can be expressed as:

\[ W = \left( \text{value of inputs} \pm \delta \right) + p \]

Then, it is clear that, although the value of inputs is a portion of the total value of commodities, it is not this magnitude, but the cost-price that enters into the value formation.

### 2.5 How does value determine production prices?

Since, as it has been seen, prices of production (in general, market prices) enter through cost-price into the formation of values, it seems that values are determined by production prices, not the opposite. How, then, does value determine production price? The next passage in the “second example”, clarifies the nature of the process of determination of production prices by values.

In this passage, Marx relates the cost-price and the value of commodities making it clear that that both magnitudes are determined in different phases of capital cycle, i.e. they are defined temporally. As commodities are not exchanged for their values but for their production prices, a divergence (“error”) occurs, but this is a “past error” (vergangener Irrthum) set at the beginning of capital circuit. So, cost-price is a given magnitude, a “premise” (Voraussetzung), of production process, while the commodity containing a surplus-value is the “result” obtained at the end of the cycle. To elaborate this distinction between “premise” and “result”, the circuit of capital can be temporally specified as a sequence of circulation time (M-C), production time (…P…) and, again, circulation time (C’-M’).

Let us suppose that there is no fixed capital, that the monetary expression of labor-time (MELT) is constant and the movements of capital through the production sphere and the two phases of the circulation sphere are accomplished successively in time [in einer zeitlichen Reihenfolge]. The duration of its stay in the production sphere forms its production time, that in the circulation sphere its circulation time. The total amount of time it takes to describe its circuit is therefore equal to the sum of its production time and its circulation time.” Capital II, p. 200; MEW 24, p. 124.


“...the movements of capital through the production sphere and the two phases of the circulation sphere are accomplished successively in time [in einer zeitlichen Reihenfolge]. The duration of its stay in the production sphere forms its production time, that in the circulation sphere its circulation time. The total amount of time it takes to describe its circuit is therefore equal to the sum of its production time and its circulation time.” Capital II, p. 200; MEW 24, p. 124.

The monetary expression of labor-time (MELT) is the amount of money in which is represented one unit of social labor-time. See Ramos [1995], [1996] and [1997]. Marx assumes explicitly a constant MELT in a tabular example of the transformation procedure, “£1 = working day”, letter to Engels, 2 August 1862, and at the beginning of the Capital III, part three, “The Law of the Tendential Fall in the Rate of Profit”, £2 = 1 working week, Capital III, p. 317; MEGA II/4.2, p.
that the first circulation phase (M-C) takes place on January 1\textsuperscript{st}, 1998. This day, capitalists advance money purchasing means of production and labor-power sold at their production prices determined by a redistribution of the surplus-value objectified over the preceding year. Social labor-time represented by these monetary production prices is thus the “premise” for a new production process (...P...), which spans from January 2\textsuperscript{nd} to December 31\textsuperscript{st}. At the end of the year, the consumption of the means of production and labor-power has “resulted” in a new commodity whose value has been determined during the production process. When workers’ living labor consumes the use-value of the means of production, the labor-time represented by their monetary production prices (constant capital) is transferred to the value of the new commodities. Workers also produce the necessary labor, given by the labor-time represented by the monetary production price of the wage-basket (variable capital). Expenditure of living labor beyond necessary labor “results” in the surplus-labor which is objectified as a surplus-value, “over and above” its “premise”, cost-price, K. As it is related to K, surplus-value assumes the mystified form of (produced) “profit” (p). So, at this point, value and surplus-value (produced profit) are quantitatively determined. The temporal process of value formation can be described by re-writing Marx’s equation for value as $K_t + p_{t+1} = W_{pt}$. The “premise” of value formation is $K_t$, given by the production prices fixed at the end of the preceding cycle. Above this “premise”, living labor creates a surplus-value during production time elapsing from $t$ to $t+1$. The “result” is $W_{pt+1}$, the value of commodity made up during the production time of year $t+1$, (the superscript “pt” stands for “production time”). At this point, the general rate of profit has also been determined as the quotient between total surplus-value objectified in the production process and total cost-price, the “premise” of capital circuit.

After production time, on January 1\textsuperscript{st}, 1999, a new circulation phase (C’-M’) is accomplished. Then, commodities are exchanged for their production prices, a quantitatively transformed form of their values given by the previously determined magnitudes, namely, $K_t$ and the general rate of profit. The “transformation” means, therefore, that commodities are not exchanged for their values determined in production time ($W_{pt+1} = K_t + p_{t+1}$), but for their production prices, a modified magnitude already determined during the production time which is manifested during circulation time, $P_{ct+1} = K_t + p’_{t+1}$, (the superscript “ct” stands for “circulation time”). Like in Marx’s equations, production prices differ from values only because the appropriated profit ($p’$) differs from the produced profit ($p$). The “transformation” brings about only a quantitative change which allows a redistribution of total profit (= total surplus value) in circulation in such a way all capitalists would obtain the general rate of profit.

It is therefore clear that the production prices that emerge at the end of any particular capital cycle cannot determine the values formed during that cycle, but, on the contrary, the latter determine the former. Production price is only an external, circulation modified form assumed for the already determined magnitude of value. Certainly, production prices fixed at the end of one capital cycle (i.e. at the beginning of the next), affect the value formation in that next cycle. However, this is only a specific consequence of the fact that social labor-time must appear as money and that this manifestation involves quantitative divergences\textsuperscript{40}. Conversely, money and, then, the different price forms (among them, production prices) are only expressions of social labor-time, so that the amounts of social-labor time represented by the paid production prices is what must enter into the cost-price of the new commodities and, then, into their values. Under the form of production price, assumed by the values at the end of a capital circuit, social labor-time becomes the determining “premise” for the value formation in the next cycle. Labor-value and monetary production prices are not absolutely “separated systems”, as purported by Tugan-Baranowsky, Bortkiewicz and other, but related facets of a unique reality: the substance

\textsuperscript{285.}
\textsuperscript{40} See footnote 15.
of value and its necessary form of manifestation whose quantitative divergences are a necessary condition for capitalists would attain the general rate of profit.

Closing the “second example”, Marx sums the relation between cost-price, value and production price up as follows:

The cost price of a commodity simply depends on the quantity of paid labour it contains, while the value depends on the total quantity of labour it contains, whether paid or unpaid; the price of production depends on the sum of paid labour plus a certain quantity of unpaid labour that is independent of its own particular sphere of production.\(^{41}\)

This only translates in words the equations contained in the passage omitted by Engels. Firstly, cost-price depends on the quantity of paid labour it contains. This paid labour (\(K_t\)) can only be the labour paid through the form of production prices at the beginning of capital circuit because commodities are no longer paid at their values. Secondly, value (\(W\)) depends on the total labour it contains “whether paid or unpaid”, so that value is formed as the sum of cost-price (paid labour, \(K_t\)) + surplus value (unpaid labour, \(m_{t+1}\)). Finally, production price (\(P\)) is the sum of cost-price (paid labour, \(K_t\)) + the amount of profit necessary for obtaining the general rate of profit (profit, \(p'_{t+1}\)).

Some authors think that the temporal process of value formation is incompatible with the determination of value by the labor-time necessary to reproduce a commodity.\(^{42}\) To discuss this, let us suppose that, at the beginning of the cycle, a flax-producing capitalist advances $100 purchasing a Ton of cotton. Over the year, the cotton is consumed and the labour-value represented by $100 is then transferred to the value of flax. At the end of the year, the capitalist sells the flax and re-purchases cotton finding that the price of cotton has fallen to $90/Ton. According to Marx, this implies that the existing stocks of cotton will be revaluated.\(^{43}\) This revaluation has been interpreted as also implying a retroactive change in the labour-value already transferred from the cotton to the flax during the preceding year so that, at the end of the year, the value transferred by the cotton would be $90, instead of $100. In other words, the labour-time paid under the form of cost-price would not be determined at the beginning of the circuit but at its end or, more precisely, the input (cotton) and output (flax) prices and values should be determined simultaneously, at the end of the cycle. This interpretation, however, does not take into account that, although the existing stocks of cotton are indeed revaluated, the cotton purchased at the beginning of the cycle has been consumed, i.e. its use-value has been destroyed during the production process. So, at the end of the cycle, that cotton has no longer exchange-value because it no longer exists and the value represented by its price was already transferred to the flax. It is neither a stock that could be revaluated nor the value advanced and transferred can be retroactively modified.\(^{44}\)

\(^{41}\) Capital III, p. 265; MEGA2 II/4.2, p. 242, lines25-30. Emphases from the Main Manuscript. There are minor modifications in the published version.

\(^{42}\) For example: “…factors which alter the prices… of commodities… will cause simultaneous changes not only in the valuation of capital in those industries which use such commodities as means of production but also in the value of the commodity output in these industries.”, Wolff, Roberts and Callari, [1982], p. 581.

\(^{43}\) Capital I, p. 317-8.

\(^{44}\) See Kliman and McGlone [1998].
3. AN EXTENSION OF THE SINGLE-TABLE TRANSFORMATION EXAMPLE

The transformation procedure can be illustrated by means of an example compatible with Tugan’s and Bortkiewicz’s. In order to do that, the single table presented in section 2.3 will be slightly modified as follows:

**Year 0: Simple reproduction without technical change**

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>v</th>
<th>K</th>
<th>m = p</th>
<th>W</th>
<th>p’</th>
<th>P</th>
<th>Q</th>
<th>W/Q</th>
<th>P/Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>170</td>
<td>30</td>
<td>200</td>
<td>30</td>
<td>230</td>
<td>40</td>
<td>240</td>
<td>100</td>
<td>2.3</td>
<td>2.4</td>
</tr>
<tr>
<td>II</td>
<td>70</td>
<td>30</td>
<td>100</td>
<td>30</td>
<td>130</td>
<td>20</td>
<td>120</td>
<td>100</td>
<td>1.3</td>
<td>1.2</td>
</tr>
<tr>
<td>Σ</td>
<td>240</td>
<td>60</td>
<td>300</td>
<td>60</td>
<td>360</td>
<td>60</td>
<td>360</td>
<td>60</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The original table has been converted into a simple reproduction schema by adding spheres 1. and 2., interpreted as producing “machines” (department I) while sphere 3 is department II, producing “corn”. Fixed capital and technical change are abstracted and it is assumed market clearing. The numbers are in money units ($), assuming that the monetary expression of labor-time is constant, $1 = 1 working day\(^{45}\), except column Q, the physical amounts of use-values, measured in natural units; W/Q and P/Q are, respectively, the unit values and the unit production prices. At the start of year 0 (M–C), capitalists advance \(K_t = $240\), purchasing 100 means of production at $2.4 each ($240\(_c\)) and 120 working days. As wage is $0.5 per working day, total variable capital is $60\(_v\), an amount that will allow the workers to purchase 50 means of consumption ($1.2 each). During the year, living labor is spent, transferring constant capital to the new commodities and generating a surplus-value (m) or profit (p) amounting to $60. Value \((W_{t+1} = K_t + p_{t+1})\) objectified in both departments is, respectively, \(W_I = $200 + $30 = $230\) and \(W_{II} = $100 + $30 = $130\). As total surplus-value is $60, the uniform rate of profit is determined as: \(60_p / (240_c + 60_v) = 20\%\). During the next circulation phase (C’–M’), commodities are exchanged for their production prices, formed as \(P^t_{t+1} = K_t + p’_{t+1}\), that is, their cost-price + the corresponding fraction of profit which would allow capitalists to obtain a 20% on their advanced capital. Values are then transformed into production prices by replacing p with p’. For department I, \(\$200*(1 + 0.2) = $240\) and, for department II, \(\$100*(1 + 0.2) = $120\). Clearly, the sum of production prices is equal to the sum of values ($360) and total surplus-value ($60) has been only redistributed in circulation. Since there is no technical change, at the end of this cycle, values and production prices are the same than those at the beginning.

To explore the effect of technical change, let us suppose that in year 1, capitalists introduce labor-saving innovations, reducing the amount of living-labor from 120 to 100 working days, ceteris paribus. The economy is, then, described as follows:

**Year 1: The effect of labour-saving innovations**

<table>
<thead>
<tr>
<th></th>
<th>c</th>
<th>v</th>
<th>K</th>
<th>M = p</th>
<th>W</th>
<th>p’</th>
<th>P</th>
<th>Q</th>
<th>W/Q</th>
<th>P/Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>170</td>
<td>25</td>
<td>195</td>
<td>25</td>
<td>220</td>
<td>33.6</td>
<td>228.6</td>
<td>100</td>
<td>2.2</td>
<td>2.286</td>
</tr>
<tr>
<td>II</td>
<td>70</td>
<td>25</td>
<td>95</td>
<td>25</td>
<td>120</td>
<td>16.4</td>
<td>111.4</td>
<td>100</td>
<td>1.2</td>
<td>1.114</td>
</tr>
<tr>
<td>Σ</td>
<td>240</td>
<td>50</td>
<td>290</td>
<td>50</td>
<td>340</td>
<td>50.0</td>
<td>340.0</td>
<td>100</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

At the end of year 0, capitalists purchase the 100 machines available for $240 and only 100 working days that cost $50. Therefore, capital advanced at the start of year 1 is lower than that advanced in year 0, releasing $10. This implies that a fraction of the corn produced in year 0 would not have buyers. Notwithstanding this, in order to maintain the simplification of market clearing, it is as-

\(^{45}\) See Ramos [1997].
sumed that the money capital released is spent by the capitalists buying an extra amount of corn (revenue). During year 1, living labour is consumed, producing a surplus-value equal to $50. Unit values are lower than in year 0, reflecting the labour-saving innovations, $W_I = $2.2$ and $W_{II} = $1.2$. The rate of profit is now equal to $50/(240_c + 50_v) = 17.2\%$ so that unit production prices are $P_I = $2.286$ and $P_{II} = $1.114$. Total value ($340$) is equal to total production price and total surplus-value ($50$) is equal to total profit. Assuming that in year 2 capitalists will purchase the same amount of machines and living labour, we can see that the reduction in the production prices provoked by the innovations will cause another release of money capital. In effect, the machines that were worth $240$ at the beginning of year 1, are now worth $228.6$, and the $100$ working days can be purchased for $46.4$, instead of $50$. This releases approximately $15$ that may be consumed by the capitalists as revenue.

It is important to note that the labour-saving innovations reduce the rate of profit measured in social labor-time. This differs from the interpretation proposed by Tugan according to which the effect of the increasing productivity is a rise in the profit rate\textsuperscript{46}. It might be alleged that this result comes from the fact that the temporally determined prices do not correspond to the equilibrium positions which would be reached after a given number of cycles if the new technique is not modified. However, there is no reason why this equilibrium position is reached before new innovations are introduced. Tugan’s result requires an instantaneous adjustment to a situation in which input and output production prices and values are identical, something that should be the outcome of a phantom-like Walrasian auctioneer but not of the dynamic of capital.

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\textsuperscript{46} See Tugan [1905], pp.174-86. This proposition is currently known as the Okishio Theorem, see Okishio [1961].


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